UPPER SIGNED EDGE UNIDOMINATION NUMBER OF A CYCLE

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Abstract: In this paper, latest concepts minimal signed edge unidominating function of a graph and upper signed edge unidomination number of a cycle is found and In addition, we determine the minimal signed edge unidominating functions with maximum weight for these graphs. The results obtained are illustrated.

Keywords: Minimal signed edge unidominating function, upper signed edge unidomination number of a graph.

Introduction: Minimal SuDF and upper signed unidomination number was defined and studied by Aruna and results on upper signed unidomination number of some corona product graphs are discussed. In this chapter, latest concepts minimal signed edge unidominating function and upper SeUDN of a graph are introduced. In addition, we determine the minimal signed edge unidominating functions with maximum weight for these graphs. The results obtained are illustrated.

Minimal Signed Edge Unidominating Function (MSEUDF)
In this section the concepts of minimal signed edge unidominating function (MSEUDF) and upper SeUDN are defined as follows:

Definition: Let $G(V,E)$ be a graph and f and g be functions from E to $\{-1,1\}$.
We say that $g < f$ if $(e) \le f(e)$ $\forall\, e \in E$, with strict inequality for at least one edge e.

Definition: Let $G(V,E)$ be a connected graph.
A SEUDF $f: E \rightarrow \{-1,1\}$ is called a MSEUDF if for all $g < f$, g is not a SEUDF.

Definition: The **upper SeUDN** of a graph $G(V,E)$ is defined as $\max\{f(E)/f \text{ is a MSEUDF}\}$.

It is denoted by $\Gamma'(G)$.
Upper SeUDN of a Cycle

Here we discuss MSEUDF of a cycle and find upper SeUDN of this graph in various cases.

Theorem: The upper SeUDN of a cycle C_n, $n \geq 3$ is

$$\Gamma'_{su}(C_n) = \begin{cases} \frac{n}{3} & \text{if } n \equiv 0(mod\ 3), \\ \frac{n+2}{3} & \text{if } n \equiv 1(mod\ 3), \\ \frac{n+4}{3} & \text{if } n \equiv 2(mod\ 3). \end{cases}$$

Proof: Let C_n be the given graph.We have the following cases.
Case 1: Let $n \equiv 0(mod3)$.
Define a function $f: E \rightarrow \{-1,1\}$ *by*

$$f(e_i) = \begin{cases} -1 & for\ i \equiv 0(mod\ 3), \\ 1 & otherwise \end{cases}$$

for all i = $1,2, \dots, n$.

This function is similar to the function defined in Case 1 of known Theorem, it is shown that f is a SEUDF.
Now we check for the minimality of f.

Define a function $g: E \rightarrow \{-1,1\}$ *by*

$$g(e_i) = \begin{cases} -1 & for\ i \equiv 0(mod\ 3)\ and\ i = 1, \\ 1 & otherwise. \end{cases}$$

Suppose $i = 1$. Then $g(e_1) = -1$.

$$\sum_{e' \in N[e_1]} g(e') = g(e_n) + g(e_1) + g(e_2) = (-1) + (-1) + 1 = -1 \neq 1.$$

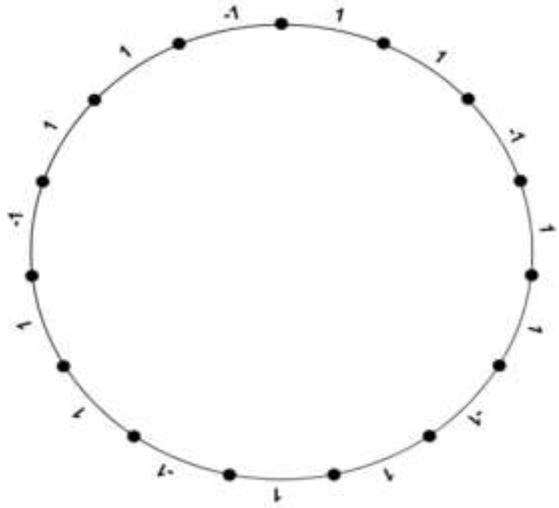
This is the case when a SEUDF fails an edge $e_1 \in C_n$ where $(e_1) = -1$ because it is in the neighborhood of the edge.
Thus g is not a SEUDF.
Since g is defined arbitrarily, there is no $g < f$ such that g is a SEUDF.
As a result, f is a MSEUDF. The only MSEUDF is f because assigning the functional values 1, −1 to the edges of C_n in any further way does not construct f any longer a SEUDF.

$$\text{Now } f(E) = \sum_{e' \in C_n} f(e') = \underbrace{(1 + 1 + (-1)) + \cdots + (1 + 1 + (-1))}_{\frac{n}{3} - times} = \frac{n}{3}$$

(Here there are $\frac{n}{3}$ groups with functional values sum as 1).

Thus $f(E) = \frac{n}{3}$.

For example, the functional values of MSEUDF f defined in this Case are given at every edge of the graph C_{15}.



Now max $\{f(E)/f$ is a MSEUDF$\} = \frac{n}{3}$, because f is the only one MSEUDF.
Therefore $\Gamma'(C_n) = \frac{n}{3}$ $n \equiv 0(mod\ 3)$.

Case 2: Let $n \equiv 1(mod\ 3)$.

Define a function $f : E \to \{-1, 1\}$ by

$$f(e_i) = \begin{cases} -1 & for\ i \equiv 0(mod\ 3), \\ 1 & otherwise \end{cases}$$

for all i = $1, 2, \ldots, n$.

This function is equal to the function defined in Case 2 of Known Theorem, it is shown that f is a SEUDF.
Now we check for the minimality of f.

Define a function $g: E \to \{-1,1\}$ by

$$(e\ )_i = \begin{cases} -1 & for\ i \equiv 0(mod\ 3)\ and\ i = 2, \\ 1 & otherwis\ . \end{cases}$$

Suppose i = 2. Then $g\ (e_2) = -1$.

$$\sum_{e' \in N[e_2]} g\ (e') = g(e_1) + g(e_2) + g(e_3) = 1 + (-1) + (-1) = -1 \neq 1.$$

This is the case when a SEUDF fails an edge $e_2 \in C_n$ where $(e_2) = -1$ because it is in the neighborhood of the edge.
Therefore g is not a SEUDF.
Since g is defined arbitrarily, there is no $g < f$ such that g is a SEUDF.
As a result, f is a MSEUDF. The only MSEUDF is f because assigning the functional values 1, −1

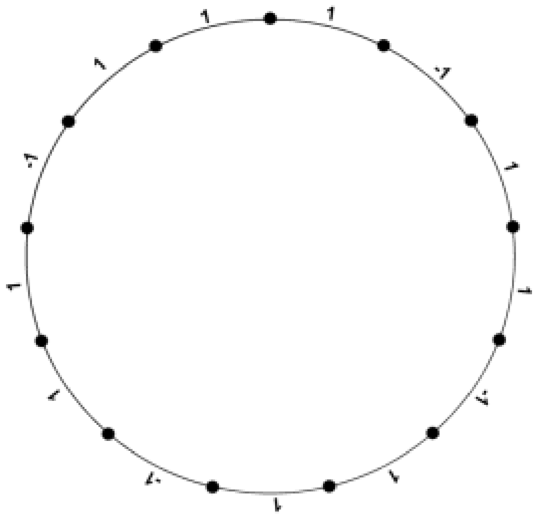
to the edges of C_n in any further way does not construct f any longer a SEUDF.

Now $$f(E) = \sum_{e' \in C_n} f(e') = \underbrace{(1+1+(-1)) + \cdots + (1+1+(-1))}_{\frac{n-1}{3}\text{ - times}} + 1$$

$$= \frac{n-1}{3} + 1 = \frac{n+2}{3}$$

(Here there are $\frac{n-1}{3}$ groups by functional values sum as 1).
Thus $f(E) = \frac{n+2}{3}$.

For example, the functional values of MSEUDF f defined in this Case are given at every edge of the graph C_{13}.



Now max $\{f(E)/f$ is a MSEUDF$\} = \frac{n+2}{3}$, because f is the only one MSEUDF.

Therefore $\Gamma'(C_n) = \frac{n+2}{3}$ when $n \equiv 1(mod\ 3)$.

Case 3: Let $n \equiv 2(mod\ 3)$.

Define a function $f: E \to \{-1,1\}$ by

$$f(e_i) = \begin{cases} -1 & for\ i \equiv 0(mod\ 3), \\ 1 & otherwise \end{cases}$$

for all i = 1,2, … , n.
This function is same as the function defined in Case 3 of Theorem 3.4.1 in Chapter 3 and it is shown that f is a SEUDF.
Now we check for the minimality of f.

Define a function $g: E \to \{-1,1\}$ *by*

$(e\ )_i = \{\begin{matrix} -1 & for\ i \equiv 0 (mod\ 3)\ and\ i = 2, \\ 1 & otherwi\ . \end{matrix}$

Suppose i = 2. Then $(e_2) = -1$.

$$\sum_{e' \in N[e_2]} g(e') = g(e_1) + g(e_2) + g(e_3) = 1 + (-1) + (-1) = -1 \neq 1.$$

This is the case when a SEUDF fails an edge $e_2 \in C_n$ where $(e_2) = -1$ because it is in the neighborhood of the edge.

Thus g is not a SEUDF.

Since g is defined arbitrarily, there is no $g < f$ such that g is a SEUDF.

As a result, f is a MSEUDF. The only MSEUDF is f because assigning the functional values –1, 1 to the edges of C_n in any further way does not construct f any longer a SEUDF.

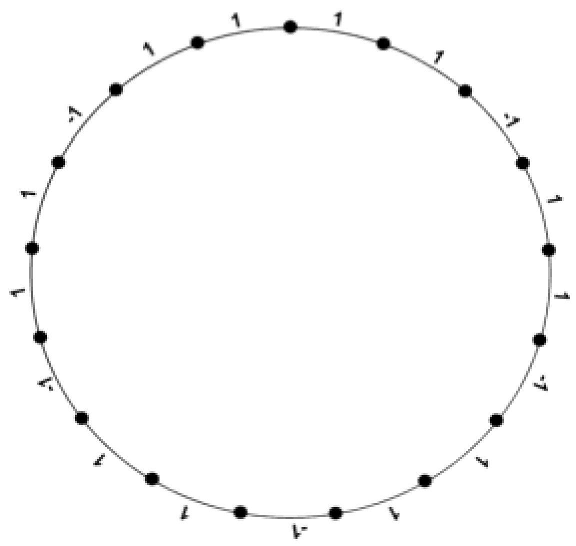
$$\text{Now } f(E) = \sum_{e' \in C_n} f(e') = \underbrace{(1 + 1 + (-1)) + \cdots + (1 + 1 + (-1))}_{\frac{n-2}{3} - times} + 1 + 1$$

$$= \frac{n-2}{3} + 2 = \frac{n+4}{3}.$$

(Here there are $\frac{n-2}{3}$ groups with functional values sum as 1).

Thus $f(E) = \frac{n+4}{3}$

For example, the functional values of MSEUDF f defined in this Case are given at every edge of the graph C_{17}.



$\Gamma'_{su}(C_{17}) = 7$

Now max $\{f(E)/f$ is a MSEUDF$\} = \frac{n+4}{3}$, because f is the only one MSEUDF.

Therefore $\Gamma'_{su}(C_n) = \frac{n+4}{3}$ when $n \equiv 2(mod\ 3)$.
Combining all three cases completes the proof of the theorem.
Thus the upper SeUDN of a cycle C_n, $n \geq 3$ is

$$\Gamma'_{su}(C_n) = \begin{cases} \frac{n}{3} & \text{if } n \equiv 0(mod\ 3), \\ \frac{n+2}{3} & \text{if } n \equiv 1(mod\ 3), \\ \frac{n+4}{3} & \text{if } n \equiv 2(mod\ 3). \end{cases}$$

Conclusion: In this paper the authors have studied minimal signed edge unidominating function, upper signed edge unidomination number of a cycle. This works throws light on further study of some other standard graphs such as complete graph etc.

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